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NUMERICAL SOLUTION OF SINGULAR INTEGRAL EQUATIONS.(U)
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One encounters difficulties in the solution of integral equations, namely, the occurrence of singularities in the kernel, and the occurrence of unknown-type singularities in the solution of the integral equation. Standard methods of approximation based on exactness for polynomials up to a certain degree are very poor, and frequently fail for functions having such singularities. The purpose of this contract was to develop methods for solving integral equations which work well in spite of the presence of singularities. This goal has been accomplished, in that the new approximation methods which were developed do work well in the presence

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Numerical Solution of Singular Integral Equations

Forward and Body of Final Report

Many problems of geophysics, meteorology, biology, electrocardiography, scattering, etcetera may be solved via the solution of integral equations. By potential theory methods, the solution of partial differential equations -- especially elliptic partial differential equations -- may be carried out via the solution of integral equations. The integral equation formulation is very appealing when it reduces the dimension of the problem by 1. This is particularly important in three dimensions, since the resulting algebra problem obtained via direct application of finite difference or finite element methods to a PDE in 3 dimensions is so large that its numerical solution is very costly, and frequently impossible.

However, one encounters new difficulties in the solution of integral equations, namely, the occurrence of singularities in the kernel, and the occurrence of unknown-type singularities in the solution (singularities in the solution may also pose a problem in the direct solution of PDE) of the integral equation. Standard methods of approximation based on exactness for polynomials up to a certain degree are very poor, and frequently fail for functions having such singularities.

The purpose of this contract was to develop methods for solving integral equations which work well in spite of the presence of singularities. This goal has been accomplished, in that the new approximation methods which we developed [1]*, [2], [3]*, [4]*, [5]* do work well in the presence of singularities, and indeed, the n-point methods converge at the rate

Here and henceforth "starred" ([]) references refer to papers supported by DAHC 04-74-G-0175 or by DAAG 29-76-G-0210.

$$(1) \quad \|\text{Error}(n\text{-point rule})\| = e^{-cn^{\frac{1}{2}d}}$$

for the case of d dimensional problems [6]*. This rate of convergence is optimal [7], in that no class of n -point methods can converge faster if the exact nature of the singularities of the solution is not known a priori.

The resulting system of algebraic equation obtained by combining the Galerkin method with the derived approximation is of relatively small order (usually less than 100 for 3d problems solved by 2d methods) and can be carried out quite effectively [8]*, [9]*, [10]*, [11]*, [12]. Indeed, although the methods [5]* lead to full matrices for the case of PDE problems, they converge so rapidly that we believe that it will be far less expensive than by finite element methods to solve 3d problems with them. At present this is only conjecture, since the methods are relatively new, and we have not yet attempted the direct solutions of 3d PDE problems with them.

We wish to mention here that Sobolev and other Russian mathematicians are also seeking optimal methods (especially quadrature rules) for solving elliptic PDE via integral equation methods [13], [14], [15]. While the optimal quadrature rules which these mathematicians identify are not known, there do exist families of known rules, which these mathematicians show to be "asymptotically optimal", i.e.

$$(2) \quad \frac{\|\text{Error}(n\text{-point optimal rule})\|_{\text{Sobolev}}}{\|\text{Error}(n\text{-point known rule})\|_{\text{Sobolev}}} \rightarrow 1 \text{ as } n \rightarrow \infty$$

where $\|\cdot\|_{\text{Sobolev}}$ denotes a suitable Sobolev norm. For a Sobolev-type optimal rule one has

$$(3) \quad \|\text{Error}(n\text{-point optimal rule})\|_{\text{Sobolev}} \sim \frac{A}{n^c} \text{ as } n \rightarrow \infty$$

where A and c are positive constants, depending upon the particular Sobolev norm. The reason for the relatively slow rate of convergence compared to (1) is that functions are allowed to compete in the process of deriving the rule via the norm (3) which have m (say) and only m derivatives at each point in the domain.

Now the solution of an elliptic PDE generally has the following properties:

- (i) It is an analytic function of each variable at each point of the interior of its domain wherever the coefficients of the DE are analytic;
- (ii) It is an analytic function of each variable on the boundary wherever the coefficients of the DE, the boundary data, as well as the boundary (i.e. the boundary is an analytic surface) are analytic.

It seems that the Soviet Mathematicians have ignored these facts, directing themselves to the completion of the Sobolev theory, rather than to the optimal methods of solutions of problems in applications.

In applications one may decompose a PDE problem into one over a finite number (usually small) of regions, in the interior of which the solution is analytic, since the location of the singularities can be determined a priori if the coefficients, the boundary data and the boundary are known. One can therefore achieve the rate of convergence (1), which is much faster than the optimal rate (3)^{*} of a Sobolev-type method. The process of subdividing the

* Once a rule is constructed which converges at the rate (3), the error generally converges at the rate (3), even if the functions to which the rule is applied are analytic. On the other hand, a rule which has the rate of convergence (1) also has the order of convergence (3) when it is applied to functions which are only m (i.e. the same number as the class for which the rule (3) was derived) times differentiable.

region in this way is described in [9]* and [11]*. We thus claim that the methods that we have developed are superior to those of the Sobolev school; they are also superior to finite difference and finite element methods, which also converge at the rate (3).

When the Galerkin method is applied to the solution of nonlinear PDE or integral equations one obtains a system of nonlinear algebraic equations. The lack of good methods for solving such equations has led us to search for methods. To this end, we have obtained a new method of computing the topological degree [16]*, [17]* based on evaluating the sign of the nonlinear function on the boundary of the region, which may be used to find regions containing a solution, as well as methods based on this to find a solution [18], [19], [20]*.

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